surface, and obtain the surface pressure distribution. Evaluate the drag and lift forces on the cylinder.

7. Derive an equation for the speed of the sound in a medium and show that for an ideal gas, $C = \sqrt{\gamma RT}$, where C is the speed of sound and γ is the ratio of specific heats. **[10]**

8. (a)Show that stress tensor is symmetric even if there is an external couple on the fluid element.

(b) If $\nabla = e_r \frac{\partial}{\partial r} + \frac{e_\theta}{r}\frac{\partial}{\partial \theta} + e_z \frac{\partial}{\partial z}$ in cylindrical coordinates, evaluate $\nabla \times \vec{V}$. **[5+5]**

Following Equations may be helpful for solving problems with cylindrical coordinates.

r momentum equation:

$$\rho\left[\frac{DV_r}{Dt} - \frac{V_\theta^2}{r}\right] = -\frac{\partial P}{\partial r} + \mu\left[\nabla^2 V_r - \frac{V_r}{r^2} - \frac{2}{r^2}\frac{\partial V_\theta}{\partial \theta}\right]$$

θ momentum equation:

$$\rho\left[\frac{DV_\theta}{Dt} + \frac{V_\theta V_r}{r}\right] = -\frac{1}{r}\frac{\partial P}{\partial \theta} + \mu\left[\nabla^2 V_\theta - \frac{V_\theta}{r^2} + \frac{2}{r^2}\frac{\partial V_r}{\partial \theta}\right]$$

Z momentum equation:

$$\rho\left[\frac{DV_z}{Dt}\right] = -\frac{\partial p}{\partial z} + \mu\left[\nabla^2 V_z\right]$$

Where, $\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2}$ and

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + V_r\frac{\partial}{\partial r} + \frac{V_\theta}{r}\frac{\partial}{\partial \theta} + V_z\frac{\partial}{\partial z}$$

Total Pages-4

M.TECH. - 1st – HPE (ME)

Fluid & Gas Dynamics

Full Marks : 70 *Time* : 3 hours

Q. No. 1 is compulsory and answer any **five** from the rest

The figures in the right-hand margin indicate marks

1. Answer all questions in brief and to the point: **[2×10]**

(i) The elements of stress tensor at a point is given as $\begin{bmatrix} 1 & 4 \\ 4 & 3 \end{bmatrix}$.Can the continuum in which this tensor exists be a fluid? Explain.

(ii) Water temperature in an open container changes from T=20^{0}C at top to T=10^{0}C at bottom. Is the water continuous with respect to temperature and density?

(iii) Is it necessary to invoke stokes hypothesis for incompressible flow? Explain.

(iv) For incompressible flow, $\frac{D\rho}{Dt} = 0$. Does it mean that ρ is constant in the flow? Explain.

(v) For a two dimensional flow $\vec{V} = 2x^2 yi - 2yx^2 j$. Does it represent a possible flow? Is velocity potential defined here?

(vi) For steady, inviscid, incompressible and irrotational flow, Bernoulli's equation is applicable only along stream line. True/False? Explain.

(Turn Over)

(vii) In most of the cases flow is viscous. In this regard, does the study of potential flow (inviscid flow) theory have any relevance?

(viii) In a fully developed laminar flow through a pipe, the shear stress on the centerline is zero. Can Bernoulli's equation be applied along the pipe centre line?

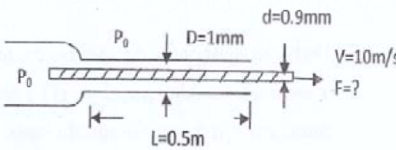
(ix) A cylinder is dragged sidewise towards right and is made to rotate clockwise in a fluid medium. What will be the direction of the Lift force i.e. upward or down ward? Explain.

(x) Define stagnation pressure for incompressible and compressible flow.

2. Show that the deformation tensor **D** is a second rank tensor. Here $d_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$. Note that transformation rules for vectors are: $a_i = l_{ij} a'_j$ and $a'_i = l_{ji} a_j$. For tensors $T_{ij} = l_{im} l_{jn} T'_{mn}$ and $T'_{ij} = l_{mi} l_{nj} T_{mn}$. **[10]**

3. Consider the flow between two horizontal porous plates y=H and y=-H, driven by an axial pressure gradient. Fluid is injected at the bottom plate with a constant velocity V_w. The fluid suction velocity at the top is also V_w. Assume that the vertical velocity is $V=V_w$ everywhere. Derive an ordinary differential equation for the axial velocity in the porous channel. Solve this differential equation to determine the axial velocity profile. You may assume that no slip boundary condition is valid for the porous plates. **[10]**

4. Coating of electric wire with insulating material is done by drawing the wire through a tubular die as shown.



The viscosity of coating material is 100 poise. Simplify the flow equations for this case and calculate the force **F** required to draw the wire. **[10]**

5. A long pipe is connected to a large reservoir that initially is filled with water to a depth of 3m. The pipe is 150mm in diameter and 6m long. Determine the flow velocity leaving the pipe as a function of time after the cap is removed from its free end. Assume the flow to be frictionless and incompressible. State any other assumptions clearly. **[10]**

6. For two-dimensional incompressible, irrotational flow, the superposition of a doublet($\psi = \frac{-\Lambda \sin\theta}{r}$), a uniform flow($\psi = Ur\sin\theta$) and a free vortex represents flow around a circular cylinder with circulation. For a clockwise free vortex($\psi = \frac{K}{2\pi}\ln r$) find the velocity field, locate the stagnation points and the cylinder